


Preparing Authority: Vincent Pugh	 G103 - A2LA Guide for Estimation of Uncertainty of Dimensional Calibration and Testing Results	Publication Date: 05/03/23
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Introduction

This document is designed to provide information on the meaning of measurement uncertainty, measurement models and equations, uncertainty budgets and the application of budgets. This guidance applies for the measurement uncertainty developed for the uncertainty claim that is provided to the customer when reporting measurement quantity values. The uncertainty provided on the calibration certificate always includes contributions from the customer's device during the test.

This guidance also applies for the measurement uncertainty developed to support the Calibration and Measurement Capability (CMC) reported on a scope of accreditation. The CMC estimate will normally be based upon the "best existing device" the laboratory can calibrate or test.

In some cases, it may be appropriate to separate the device under test contribution from the other contributors to the CMC budget¹. See [G118 - Guidance for Defining the Scope of Accreditation for Calibration Laboratories](#) for more information.

The Meaning of Uncertainty

Almost everyone makes measurements and almost everyone can ascribe some measure of uncertainty to this measurement. If we measure the area of a room by stepping off the length and width, we know that the number we get is not as "good" as if we had used a precision tape measure. We also know that if we are measuring the diameter of a crankshaft pin, we need a more accurate number than if we are measuring the diameter of a fence post to decide how big to dig the hole.

Sometimes we even make uncertainty statements along with our measurements. If asked how good a particular measurement is, we may reply "right on" or some other colloquialism. If we know we just guessed, we may say "about six inches" or whatever the number was. We also know that sometimes it is economically feasible to use better equipment and be more accurate in our measurements. If we are only interested in the approximate amount of carpet we need for a room it is okay to step off the dimensions. But if we are buying expensive carpet we may want a more accurate number than we would get by stepping off the area. If we use a precision tape, take care in making the measurement, and check our arithmetic, we can be confident that we have a number good enough for our purpose. That is what uncertainty statements are about – a statement as to the confidence that we have in the measured value.

The same reasoning applies for scientific measurements or for measurements made in a calibration or testing laboratory. National laboratories have for many years insisted that an uncertainty statement accompany all measurements. However, not all laboratories computed the uncertainty in the same manner nor reported the uncertainty with the same confidence level. For this reason, the [GUM: Guide to the Expression of Uncertainty in Measurement](#) was written so that an uncertainty statement made by one laboratory would have same meaning as one from any other laboratory.

Measurement Equation

The GUM begins by telling us to describe the measurement problem, generally by writing the equation for determining the value we want to find. If it is the volume of a cylinder, for example, we write $V = \pi r^2 h$. If it is the area of a floor, we write $A = L \times W$. If we were attempting to find the diameter of a cylinder by weighing the cylinder, measuring its length, and looking up the density in a handbook, the equation would be a little longer. In

¹ [ILAC P14: ILAC Policy for Uncertainty in Calibration](#)

dimensional measurements, if we want the diameter of an object, we measure the diameter; if we want the length, we measure the length.

This makes computing uncertainty values for dimensional measurements much easier than it is for some other measurements. If we wish, we can write an equation² for a dimensional measurement in the form

$$Y = X + \sum_i C_i \quad (1)$$

where Y is the corrected value, X is the measured value, and the C_i are corrections to be added to the measured value. It is customary to use small letters in the equation when we are using actual measured values.

For example, the equation for measuring the distance between two points using a tape measure might be written as

$$y = x + c_1 + c_2 \quad (2)$$

where y is the corrected length, x is the measured length, c_1 is a thermal correction for the tape not being at 20°C, and c_2 is a correction for the uncertainty in the tape taken from a calibration report.

In most dimensional measurements the equation for determining the desired dimension is so simple that there is no real need to write an equation as described in the GUM.

Uncertainty Budgets

An uncertainty budget is a way of documenting the estimate of the uncertainty of a measurement. An uncertainty budget can consist of only two columns (see Table 1). In the first column we list the sources of uncertainty, that is, those factors that might have caused errors in our measured values. In dimensional metrology some of these sources are the uncertainty of the master, repeatability, cosine error, elastic deformation, error caused by the temperature not being at the standard temperature of 20°C, errors caused by the test item being at a different temperature than the master, an error in the thermometer measuring the room or part temperature, scale errors in the measuring instrument, errors due to fixturing, and imperfect geometry of the part. The list should contain all quantities that might contribute to the uncertainty of the measured value³. We may find that some (perhaps most) of the sources of uncertainty are not significant. As an example, if we are comparing gage blocks of the same material the uncertainty due to deformation will be relatively small, but if we were measuring a ball or a plastic cylinder, it might not be insignificant. Later, in another uncertainty budget, when we have determined that a particular parameter is insignificant, we can then omit it from the budget.

The second column of the uncertainty budget will contain the standard uncertainty of the error source. It is the mathematics in the second part that daunts many people, but understanding the measurement process and identifying the sources of error is the hardest part in developing an uncertainty budget. The mathematics for developing uncertainty budgets for dimensional measurements is easy in most cases.

Distributions

Most people have heard of a normal curve or normal distribution, sometimes called the bell curve. If we take a thousand readings of a gage block using a comparator with a resolution of 0.1 microinch and graph the results, we will get a bell-shaped curve centered about the mean of the readings. This is a normal distribution. Ninety-five percent of the readings will fall within plus-or-minus two standard deviations of the mean.

² GUM calls this a “mathematical model”.

³ See A2LA [R205 - Specific Requirements - Calibration Laboratory Accreditation Program](#) for additional guidance.

But not all distributions are normal. If we measure the deviations from nominal for a set of Grade 2 gage blocks, the observed curve will not be a normal distribution. This is because the manufacturing process for producing gage blocks is to sort the blocks with those meeting Grade 1 requirements labeled Grade 1, and those passing Grade 2 labeled as Grade 2, etc. Therefore, the distribution of the graph of the deviations from nominal will show few blocks at the center of the graph and more toward the edges. This is called a U distribution.

A digital micrometer that reads to 0.0001in will round the actual number inside the instrument to the nearest 0.0001 in. If the instrument is capable of reading to 10 μ in, we are throwing away the last digit. The last digit that we can't see is equally likely to be any number from 0 to 9. This is an example of a rectangular distribution. The standard uncertainty is found by dividing half of the resolution by the square root of three. In the example of the micrometer with 100 μ in resolution the standard uncertainty of the resolution (u_{Res}) is given by

$$u_{\text{Res}} = \frac{50 \mu\text{in}}{\sqrt{3}} \cong 29 \mu\text{in}.$$

The triangular distribution is used when it is known that most of the values are likely to be near the center of the distribution. The standard uncertainty is found by dividing the half-interval by $\sqrt{6}$. For example, suppose that the lab temperature is controlled by a continuous cooling/variable re-heat system in such a way that the actual temperature is always near the center of the range $20^{\circ}\text{C} \pm 2^{\circ}\text{C}$. The half-interval of the allowed temperature range is then 2°C and the standard uncertainty is given by $u_{\text{Temp}} = \frac{2^{\circ}\text{C}}{\sqrt{6}} \cong 0.82^{\circ}\text{C}$.

(If the lab temperature were controlled simply by an on/off air conditioning system, then the U-shaped distribution would be more appropriate since the lab temperature is more likely to be near the extremes of the allowed temperature range. The divisor for the U distribution is $\sqrt{2}$ so if this type of environmental control were in place we would obtain as the standard uncertainty, using the control limits above, $u_{\text{Temp}} = \frac{2^{\circ}\text{C}}{\sqrt{2}} \cong 1.4^{\circ}\text{C}$.)

The GUM tells us to use the proper distribution if we know the distribution. If the type of distribution is not known, the GUM practice is to assume that it is a rectangular distribution.

Sample Uncertainty Budget for XX Master Setting Plug

Our first sample uncertainty budget is for the measurement of a XX master setting plug and is shown in Table 1. The measurement is made by comparing the cylinder to a gage block using a gage block comparator. The comparator was in a room maintained at $20^{\circ}\text{C} \pm 1^{\circ}\text{C}$.

Type A and Type B Estimates of Uncertainty

Note that there is either an A or a B following each source of uncertainty. Type A uncertainty estimates are those determined using statistics in the current measurement process. Type B uncertainty estimates are those determined by any other means⁴. For instance, one would usually list the uncertainty due to our lack of knowledge of the correct thermal coefficient of expansion of a material as a type B uncertainty estimate since it may be obtained from a handbook. However, if, in our measurement process, we measured the part at different temperatures, some above and some below 20°C , then it would be a type A uncertainty estimate since it was obtained by the statistical evaluation of data.

⁴ Some people equate type A uncertainty estimates with “random” uncertainties and type B estimates with “systematic” uncertainties. This usage is incorrect: “A” and “B” refer to how the estimate was made, not to the nature of the uncertainty contributor itself.

Uncertainty of the Master Gage Block

We are measuring the cylinder by comparing it to a 0.5-inch gage block. We can get the standard uncertainty for the gage block from the calibration report. We will use the reported value of the block in computing the difference between the block and the cylinder. This reported value has an uncertainty associated with it. It should be reported on the calibration certificate as an expanded uncertainty representing approximately a 95% confidence level ($k=2$, or “2 sigma”). To get one standard uncertainty ($k=1$, or “one sigma”), we take half of the reported uncertainty. In our case the reported uncertainty was 4 microinches at the 95% level of confidence so the standard uncertainty is 2 microinches.

Repeatability

Repeatability is defined as the closeness of the agreement between the results of successive measurements of the same item carried out under the same conditions. Repeatability is one source of uncertainty in our measurement: if we have enough resolution in our measuring instrument, and if we take numerous readings, the readings will not all be the same. The standard deviation of the readings is used as a measure of repeatability.

In this example, we found the repeatability by zeroing the comparator on the gage block, measuring the plug, and repeating the set-up and measuring process 30 times⁵.

To get the standard deviation, find the average of the 30 readings. Then list the deviation of each reading from the average, square each of these numbers and take the summation of all the squares. Divide the sum of the squares by one less than the number of readings, take the square root of the result, and we have the standard deviation. Expressed as an equation, “ s ”, the standard deviation⁶ is defined as:

$$s = \sqrt{\frac{\sum_{i=1}^n (\bar{x} - x_i)^2}{(n-1)}} \quad (3)$$

The standard deviation of the mean or average value obtained from our 30 readings is obtained by dividing the standard deviation by the square root of the number or readings taken.

$$\bar{s} = s / \sqrt{n}$$

Because we calculated the standard deviation using statistics in the current process, we call it a type “A” uncertainty estimate. In our example, we found that the mean standard deviation is 2 μin ⁷.

Note: If using a calculator, care should be taken in how the numbers are expressed⁸. It is often the case that calculators cannot correctly evaluate small values of standard deviation.

⁵ It would not do just to set the comparator once and then retract and release the measuring plunger 30 times. It’s important to be aware of what parts of the process the repeatability study is supposed to cover.

⁶ Sigma (σ) is used for the “population standard deviation” when we have readings from the entire population. When we take only a sample of the population, we use s instead of sigma to represent the sample standard deviation. Calculators will use the symbol “ s ” or “ σ_{n-1} ” for this function. Microsoft Excel uses the “STDEV” function to find the sample standard deviation.

⁷ UKAS M3003, The Expression of Uncertainty and Confidence in Measurement, 2012, pp. 18-19.

⁸ UKAS M3003, The Expression of Uncertainty and Confidence in Measurement, 2012, pp. 78-79.

For example, suppose we try to use a calculator to find the standard deviation of these four numbers: 0.500 000, 0.500 003, 0.500 007, 0.499 998. The mean of these four numbers is 0.500 002. The correct standard deviation is 0.000 003 9. But most hand-held calculators will display an error message, or display an incorrect non-zero value, or else return 0 as the standard deviation, which is obviously incorrect. (The standard deviation can be zero if and only if all of the numbers in the data set are the same.)

If we examine the equation for the standard deviation, we can see how these errors result. Each data point is first subtracted from the mean. Each of these differences can be quite small. For example $(0.500\,002 - 0.500\,000) = 0.000\,002$, which is then squared. In this example, the square of the difference is 0.000000000004, which is beyond the internal resolution of most calculators.

The solution to this problem is to use only the last few significant figures in the data set. For example, we could have written those four numbers as 0×10^{-6} , 3×10^{-6} , 7×10^{-6} , -2×10^{-6} after subtracting 0.5. Ignoring the “ $\times 10^{-6}$ ” for the moment, we can write the data set as 0, 3, 7, -2. A hand-held calculator will now be able to return the correct standard deviation of $3.9(\times 10^{-6})$.

Computers and spreadsheet programs are not immune from this effect so suitable checks should be devised to look for any such effects when using spreadsheets.

Scale Error

The comparator used to make the comparison between the gage block and the gage may have a scale error. By looking at the calibration records for the comparator, we see that the largest error at recalibration was five percent over the usable range. We will use this uncertainty budget for all 0.5 in plug gages that we measure on this instrument so we estimate the maximum deviation we might see between the block and the gage. A XX plug gage is unlikely to deviate more than 40 microinches from the gage block, so we will consider 40 μin as the worst case deviation. Five percent of the worst case reading gives a worst case error due to scale error of 2 μin . We now need to convert the worst case deviation to one standard uncertainty. If we know something about the distribution, there are formulas to make the conversion. The usual practice if we know nothing about the distribution is to consider it a rectangular distribution and divide by the square root of 3 to get one standard uncertainty. In our case, 2 divided by the square root of three gives about 1.2 microinches.

Elastic Deformation

We are using a single probe indicator with the force of the probe set at two ounces (approximately 0.5 N). Because we are using a steel gage block to measure a steel cylinder, the elastic constants will be nearly the same, but there is a slight difference between a ball to a flat and a ball to a cylinder. We can calculate the difference using the equations given in the paper by Puttock and Thwaite (see References). The equations show that the difference is a negligible amount of about $\pm 0.1\,\mu\text{in}$. We'll treat this as a rectangular distribution and divide by $\sqrt{3}$ to obtain the standard uncertainty of 0.06 μin .

Force Setting

We are using a single probe indicator with the probe setting at a nominal two ounces. The actual setting may be off. An examination of our records shows that at recalibration it has never differed more than 10% from the nominal value so we can use this as the worst case. Changing the force by 10% does not change the elastic deformation correction significantly so we list 0 as the uncertainty due to the error in the force setting. (This is true because the gage block and the gage are made of the same material; it would not necessarily be true if the gage block and cylinder were made of different materials.)

Effects of Temperature

There are two possible sources of error related to temperature. The room may not be at exactly the standard temperature for dimensional measurements (20°C). If the master and the test have different coefficients of thermal expansion and no corrections are made, an error will result. It is also likely that the master and the test are not at exactly the same temperature, which is another source of error.

Coefficient of Thermal Expansion

We stated that the laboratory is maintained at 20°C ±1°C. However, the thermometer doesn't measure the temperature exactly so there is some uncertainty about the temperature indicated by the thermometer. Assuming the room thermometer is good to ±0.1°C, the temperature of the room could be anywhere between 18.9°C and 21.1°C. If the block and cylinder are at exactly the same temperature, the temperature could deviate from the standard 20°C by ±1.1°C.

Coefficients of thermal expansion are not known exactly. Even when the master and the test item are both steel and both are at the same temperature, possible differences in the thermal coefficient of expansion can cause uncertainties in the measurement. It is reasonable to assume a difference in coefficients of expansion of 1.5 ppm/°C between the steel in the master and the steel in the test gage. If the master and test were 1.1°C from nominal, the maximum error would be:

$$\Delta L = L * \Delta T * \Delta \alpha \quad (4)$$

where ΔL is the error in length, L is the nominal length, ΔT is the deviation from 20°C, and $\Delta \alpha$ is the difference between the coefficients of expansion of the cylinder and the coefficient of expansion of the gage block. For a 0.5-inch interval, we have $\Delta L = (0.5 \text{ in})(1.1^\circ\text{C})(1.5 \text{ ppm}/^\circ\text{C})$ or 0.82 μin .

It is unlikely the worst case will always happen so we will treat it as a rectangular distribution and divide by the square root of three to get 0.48 microinch as the standard uncertainty caused by errors due to uncertainty in the coefficient of thermal expansion.

Differences in Temperature

In this case the operator wore gloves, we did not measure the temperature, and the difference in temperature between the master and test gages was estimated not to exceed 0.1°C. If there is a difference in temperature of 0.1°C between the master and the test gages, it will cause an error of (see equation 4) $\Delta L = (0.5 \text{ in})(0.1^\circ\text{C})(11.5 \text{ ppm}/^\circ\text{C}) = 0.58 \text{ microinch}$.

We will again assume a rectangular distribution⁹ and divide by the square root of three to get a standard uncertainty of 0.33 microinch.

Table 1. Uncertainty Budget for 0.5-inch XX Plain Plug Gage

⁹ We could also have used a triangular distribution here. The most likely value for the difference in temperature between the master and test gages is 0. The least likely, but still possible, is ± the allowed temperature variation in the room (i.e., one part is as hot as the room gets, the other part is as cold as the room gets). In a situation like this, the lab can allow for large temperature differences while noting quantitatively that large differences don't happen very often by using the triangular distribution. Assuming a triangular distribution in this case yields a standard uncertainty of 0.24 μin ($0.58 \mu\text{in}/\sqrt{6}$) and a final expanded uncertainty of 6 μin so, in this case, assuming a triangular or rectangular distribution makes no difference to the final answer. The issue is a philosophical one having to do with different engineering judgements and opinions on how best to handle the statistics of temperature effects.

Source of Uncertainty	Standard Uncertainty (μin)
Master gage block uncertainty (B)	2.0
Repeatability (A)	2.0
Scale error (B)	1.2
Elastic deformation (B)	0.06
Force setting (B)	0
Coefficient of thermal expansion (B)	0.48
Part and master temperature difference (B)	0.33
Combined standard uncertainty: $u_c = \sqrt{2^2 + 2^2 + 1.2^2 + 0.06^2 + 0.48^2 + 0.33^2}$	3.13
Expanded uncertainty, $U = 2u_c$ ($k=2$)	6.26

The uncertainty estimate of 6.26 μin represents an expanded uncertainty expressed at approximately the 95% level of confidence using a coverage factor of $k = 2$. It is standard practice to round uncertainty estimates up, so most practitioners would round the estimate of 6.26 μin to 7 μin instead of to 6 μin . In no case is it appropriate to report uncertainty estimates to more than two significant figures. (However, one should retain extra significant figures in intermediate calculations so as to minimize round-off errors.)

An examination of the uncertainty budget shows that the dominant sources of uncertainty are the uncertainty of the master gage block and the repeatability. Having better temperature control would not help much.

Sample Uncertainty Budget for a 0.5-Inch Plain Ring Gage

A sample uncertainty budget for the measurement of a 0.5 in plain ring gage is shown in Table 2.

In our second example of an uncertainty budget, we have changed the format of the budget by adding additional columns. We gave the type of uncertainty estimate (A or B) in a separate column instead of in parentheses after the source of uncertainty, we have added the type of distribution, and we have added a column called “variance”, which is just the standard uncertainty estimate squared. In order to calculate the combined standard uncertainty, u_c , it is necessary to square each number; therefore, it requires no additional work to list the squared value in a column. By looking at the variance, it is easy to see the sources of uncertainty that dominate the uncertainty budget.

We have the same sources of uncertainty as we had for the plug gage in Table 1, except we added probe misalignment as a potential additional error source. We have changed the temperature from Celsius to Fahrenheit as that was the common way of expressing the temperature in this particular laboratory.

The measurements were taken in a laboratory maintained at $68^\circ\text{F} \pm 2^\circ\text{F}$. The uncertainty in the measurement of the temperature was 0.2°F . The measurements were made by comparing the ring to a stack of gage blocks (with end caps) using a comparator-type instrument. The gages were allowed to stabilize to reach temperature equilibrium. We estimate that the master and the part do not differ in temperature by more than 0.2°F .

Gage Blocks

The 5 μin in the outer limit column for gage blocks came from the calibration report. We used the actual reported value of the blocks to set the internal comparator. The estimated uncertainty given on the calibration report at a 95% confidence level ($k=2$) was 5 μin . To get one standard uncertainty, we divided by two to get 2.5 μin .

Repeatability

The repeatability was estimated from the standard deviation of 30 readings obtained by repeating the complete set-up and measuring procedure 30 times.

Uncertainty in Coefficient of Thermal Expansion

The uncertainty caused by the temperature not being at exactly 68°F and our lack of knowledge about the coefficient of thermal expansion (CTE) was computed using equation 4:

$$\Delta L = (0.5 \text{ in})(2.2^\circ\text{F})(0.9 \text{ ppm}/^\circ\text{F}), \text{ or } 0.99 \text{ microinch.}$$

Uncertainty Caused by Test Part and Master Being at Different Temperatures

The uncertainty caused by the part and the master not being at the same temperature was computed using equation 4:

$$\Delta L = (0.5 \text{ in})(0.2^\circ\text{F})(6.5 \text{ ppm}/^\circ\text{F}), \text{ or } 0.65 \text{ microinch.}$$

Probe Misalignment

If both probes are not in line with the axis of measurement, a ring will measure smaller than it would otherwise. The error is larger for a small ring and becomes almost insignificant as the rings get over an inch in diameter. Discussions with NIST personnel and instrument manufacturers indicate that a worst-case estimate of the error due to probe misalignment is 2 μin for a 0.5 inch ring gage. To convert the worst-case error to one standard uncertainty, we assume a rectangular distribution and divide by the square root of three.

Comparator Calibration

The history of the calibration for the comparator showed the worst-case error to be 3% of full scale. The gage block will always be within 100 microinches of the ring so the worst-case error would be 3 microinches.

The uncertainty for the measurement of the 0.5 in ring gage is 8 μin , which represents an expanded uncertainty expressed at approximately the 95% level of confidence using a coverage factor of $k=2$.

By looking at the variance column, we can see that the gage blocks are the largest contributor to the uncertainty. We can lower the uncertainty by having the blocks calibrated by a laboratory giving a lower uncertainty.

The temperature is relatively unimportant for a 0.5 inch ring but let us do another uncertainty budget for a 10-inch ring under the same conditions (see Table 3).

Table 2. Uncertainty Budget for 0.5 in Plain Ring Gage

Uncertainty Source	Estimate (μin)	Type	Distribution	Divisor	Standard Uncertainty (μin)	Variance (μin^2)
Gage blocks	5	B	Normal	2	2.5	6.25
Repeatability	1.8	A	Normal	1	1.8	3.24
Uncertainty of CTE	0.99	B	Rectangular	$\sqrt{3}$	0.57	0.32
Master/part temperature difference	0.65	B	Rectangular	$\sqrt{3}$	0.38	0.14
Probe misalignment	2	B	Rectangular	$\sqrt{3}$	1.15	1.32
Comparator calibration	3	B	Rectangular	$\sqrt{3}$	1.73	3
Sum of the variances	14.27					
Combined standard uncertainty (u_c , equal to the square root of the sum of the variances)	3.78					
Expanded Uncertainty U ($= 2u_c$; $k=2$)	$7.56 \approx 8$					

The uncertainty of the gage blocks is larger as expected; the repeatability is also larger as might be expected, but the largest increase is in temperature effects. Both terms related to temperature increase by a factor of 20. The uncertainty is directly proportional to the diameter of the ring. The dominant source of uncertainty is now the uncertainty of the coefficient of thermal expansion.

Table 3. Uncertainty Budget for 10 inch Plain Ring Gage

Uncertainty Source	Estimate (μin)	Type	Distribution	Divisor	Standard Uncertainty (μin)	Variance (μin ²)
Gage Blocks	10	B	Normal	2	5	25
Repeatability	3.2	A	Normal	1	3.2	10.24
Uncertainty of CTE	20.0	B	Rectangular	√3	11.5	132.2
Master/part temperature difference	3.25	B	Rectangular	√3	1.88	3.52
Probe misalignment	0.5	B	Rect	√3	0.29	0.08
Comparator calibration	3	B	Rect	√3	1.73	3
Sum of the variances	174.04					
Combined standard uncertainty	13.19					
Expanded Uncertainty U ($k=2$)	26.39 ≈ 27					

The uncertainty for the measurement of the 10 in ring gage is 27 μin, which represents an expanded uncertainty expressed at approximately the 95% level of confidence using a coverage factor of $k=2$.

Measurement Uncertainty Expressed as a Function of Length

To express the measurement uncertainty for ring gages from 0.125 in up to 10 in, we would have to divide up the rings into different categories based on the size of the rings, each category having the same measurement uncertainty. Alternatively, we could state the uncertainty in the form of a single equation that describes the measurement uncertainty over the entire range. That is, if the function is nearly linear, it is simpler to put it in the form of an equation:

$$U = b + mL. \quad (5)$$

This is just the equation of a straight line where b is the y intercept (the point where the line intercepts the y -axis), and m is the slope of the line.

For example, the best uncertainty for a 0.5-in ring was found to be 8 μin and the best uncertainty for a 10-in ring was found to be 27 μin. This information gives us two points on a straight line ($L_1=0.5$ in, $U_1=8$ μin; $L_2=10$ in, $U_2=27$ μin). This can be expressed in the form of an equation for the rings in the range (0.5 to 10) inches as $U = (7 + 2L)$ μin where L is the nominal diameter of the ring in inches¹⁰. This linear approximation works only between the two end points; if we try to apply the equation to points outside of the range (in this example, [0.5 to 10 in]), we will end up underestimating the uncertainty.

¹⁰ If L_1 is the lower end of the length range and U_1 is the uncertainty at L_1 , and if L_2 is the upper end of the length range and

U_2 is the uncertainty at L_2 so that $U_1 < U_2$ and $L_1 < L_2$, then the constants m and b are $m = \left(\frac{U_2 - U_1}{L_2 - L_1} \right)$ and

$b = U_1 - \left(\frac{U_2 - U_1}{L_2 - L_1} \right) L_1$ or $b = U_1 - mL_1$.

Indicating Measuring Instruments

Unlike gage blocks, rings, and other dimensional gauges, micrometers and calipers provide direct measurement indication. The repeatability and resolution of such instruments are not inherent components of the calibration process but of the measuring instrument itself.

Sample Uncertainty Budget for a (0 to 36) Inch Micrometer

The measurements were made in a laboratory controlled to $20^{\circ}\text{C} \pm 2^{\circ}\text{C}$. The thermometer that measures the room temperature has an uncertainty of $\pm 0.25^{\circ}\text{C}$. The comparison was made by measuring gage blocks.

Master Values

For this measurement we used B89 Grade 0 blocks. We did not correct for the measured values of the blocks. We only know that the blocks were within tolerance. We took as the uncertainty the root sum square of the tolerance for 3 Grade 0, 12-inch blocks (28 microinches), which gives an uncertainty of 49 μin . We treat this as a rectangular distribution and divide by the square root of three to get a standard uncertainty of about 28 μin .

Resolution

The resolution of the digital micrometer is 100 microinches. To obtain the standard uncertainty we divide the resolution by two and then by the square root of three, or $100/2\sqrt{3} = 29$

Repeatability

The repeatability was computed from six readings taken on a gage block combination consisting of three 12-inch gage blocks fastened together with tie rods. The blocks were taken apart and rewung between each reading. The standard deviation for the repeatability on the 36-inch combination was 620 microinches.

We have added a column to the table labeled “degrees of freedom.” The number of degrees of freedom is an indication of how much information an uncertainty estimate is based on. The larger the number of degrees of freedom, the more information was available. For an uncertainty estimate taken as the standard deviation of n measurements, the number of degrees of freedom is $n - 1$ (which, not coincidentally, is also the denominator in the formula for the standard deviation). When the number of degrees of freedom is small, as in this case ($n - 1 = 5$), we can’t use a coverage factor $k = 2$. We refer instead to the “t table” (see Appendix A) to find out which coverage factor to use at a given level of confidence. In this example, we have 5 degrees of freedom and the coverage factor at the 95% level of confidence is $k = 2.57$.

It is customary to take the number of degrees of freedom associated with a type B uncertainty estimate as “infinity”. When using spreadsheets, some large number such as 100 or 1000 is used to represent infinity.

Temperature

The room is maintained at $20^{\circ}\text{C} \pm 2^{\circ}\text{C}$. Assuming the room thermometer is good to $\pm 0.25^{\circ}\text{C}$, the room temperature could be anywhere between 17.75°C and 22.25°C . It is also possible that the block and the micrometer may not be at the same temperature. All three factors can contribute to the uncertainty of the calibration. In our case the operator wore gloves, we did not measure the temperature, and the difference in temperature between the block and the micrometer was estimated not to exceed 0.5°C .

If the blocks and micrometer are at the same temperature, the temperature could deviate from the standard 20°C by 2.25°C . Even when both are at the same temperature, possible differences in the thermal coefficient of

expansion can cause uncertainties in the measurement. It is reasonable to assume a difference in coefficient of expansion of $1.5 \times 10^{-6}/^{\circ}\text{C}$ between the steel in the gage block and the steel in the micrometer. If the block and micrometer were 2.25°C from nominal, the maximum error would be $\Delta L = (36)(2.25)(1.5)(10^{-6})$ or 122 microinches.

If the micrometer is as much as 0.5°C warmer than the gage block stack, then the possible error could be $\Delta L = (36)(0.5)(11.5)(10^{-6})$ or 207 microinches. But it is unlikely the worst case will always happen so we will treat both of the above as a rectangular distribution and divide by the square root of three to get one standard uncertainty.

Table 4. Uncertainty Budget for Calibrating a (0 to 36) Inch Micrometer

Uncertainty Source	Estimate (μin)	Degrees of Freedom ν_i	Type	Distribution	Divisor	Standard Uncertainty (μin)	Variance μin^2
Gage Blocks	49	∞	B	Rectangular	$\sqrt{3}$	28.3	801
Resolution	100	∞	B	Rectangular	$2\sqrt{3}$	28.9	835
Repeatability	620	5	A	Normal	1	620	384400
Uncertainty of coefficient of thermal expansion	122	∞	B	Rectangular	$\sqrt{3}$	70.4	4956
Master/part temperature difference	207	∞	B	Rectangular	$\sqrt{3}$	119.5	14280
Summation	405272						
Combined standard uncertainty, u_c	636.6						
Expanded Uncertainty, $U = 2.57u_c$	$\cong 1700$						

For the purposes of CMC, it is acceptable to separate out the device contribution. In this case, separating out the resolution contribution would not be advisable since the resolution is a minor contributor to the overall uncertainty of the measurement.

If we want to develop an equation for all micrometers, we can develop an uncertainty budget for a 0 – 1 in micrometer, assume the equation is linear between 1 and 36 inches, and write an equation in the form of $U = a + bL$. The typical uncertainty for a (0 to 1) in micrometer is about 70 microinches so the equation might be $U = (26 + 44L) \mu\text{in}$.

Sample Uncertainty Budget for a (0 to 6) Inch Digital Caliper

The caliper was calibrated using gage blocks in a room controlled to $\pm 5^{\circ}\text{F}$. The caliper reads to the nearest 0.0005 inch. It was estimated that the temperature of the caliper and gage blocks did not differ by more than 1.0°F .

Gage Blocks

We did not use the actual values of the gage blocks; we only assumed they were within B89 Grade 0 tolerance. Our worst-case error is the tolerance of Grade 0 blocks (16 µin).

Repeatability vs. Resolution

Repeatability is defined as the closeness of the agreement between the results of successive measurements of the same item carried out under the same conditions. If we have enough resolution in our measuring instrument, and if we take numerous readings, the readings will not all be the same. Resolution is defined as the smallest increment that can be detected by the instrument.

We measured the repeatability of the (0 to 6) inch caliper by taking 31 readings and got a standard deviation of 0.00012 inch (120 microinches).

The resolution of the instrument is one half of the least count of the instrument. Dividing by the square root of three gives us one standard uncertainty. That is, the standard uncertainty is $500/(2\sqrt{3})$, which equals 144 microinches. The instrument cannot read any better than the resolution.

It is common practice¹¹ to take the larger of the repeatability or the resolution for the budget. However, in this case the contributors are approximately equal in size and, both should be included.

Uncertainty in Coefficient of Thermal Expansion

The uncertainty caused by the temperature not being at exactly 68°F and our lack of knowledge about the coefficient of thermal expansion was computed using equation 4:

$$\Delta L = (6 \text{ in}) (5.0^\circ\text{F}) (0.9 \times 10^{-6}/^\circ\text{F}) \cong 27 \text{ microinches.}$$

Uncertainty Caused by Test Part and Master Being at Different Temperatures

The uncertainty caused by the part and the master not being at the same temperature was computed using equation 4:

$$\Delta L = (6 \text{ in}) (1.0^\circ\text{F}) (6.5 \times 10^{-6}/^\circ\text{F}) \cong 39 \text{ microinches.}$$

The dominant component in this uncertainty budget is the resolution. If resolution is the only significant contribution to the budget, the expanded uncertainty for digital instruments may be reported as $U = 0.6R$, or $U = R/\sqrt{3}$, where R is the resolution (however, see the section below titled “When k Does Not Equal 2”)

For cases when the uncertainty budget is developed to support the CMC claims on a scope, it may be appropriate to separately identify instrument-particular uncertainty components such as resolution that dominate the budget^{12,13}.

¹¹ See, for example, ISO/TS 14253-2

¹² [ILAC P14: ILAC Policy for Uncertainty in Calibration](#)

¹³ M3003, The Expression of Uncertainty and Confidence in Measurement, 2007, p. 31.

Table 5. Uncertainty Budget for Calibrating (0 to 6) Digital Caliper

Uncertainty Source	Estimate (μin)	Degrees of Freedom	Type	Distribution	Divisor	Standard Uncertainty (μin)	Variance (μin ²)
Gage Blocks	16	∞	B	Rectangular	√3	9.24	85.4
Resolution	500	∞	B	Rectangular	2√3	144	20736
Repeatability	120	30	A	Normal	1	120	14400
Uncertainty of CTE	27	∞	B	Rectangular	√3	15.6	243
Master/part temperature difference	39	∞	B	Rectangular	√3	22.5	506
Summation	35970						
Combined standard uncertainty, u_c	190						
Expanded Uncertainty, U $= 2u_c$	380~ 400						

Sensitivity Coefficients

The GUM tells us to write the equation to determine the measurand (the quantity we want) in relation to the variables that may affect the measurement. We stated earlier that in most cases it is not necessary to write this equation. This is not always the case, however. We will give one example where it is necessary to express the equation to obtain the measurand.

The pitch diameter of a thread gage is defined as the diameter of the cylinder that divides the thread and thread spacing into equal parts. The pitch diameter is measured by placing three (sometimes two) wires in the thread grooves, choosing wires that touch near the pitch diameter cylinder. The measurement is made by measuring over the wires and subtracting a “C” correction to obtain the pitch diameter. The equation for finding the pitch diameter is:

$$PD = M_w - \left(w(1 + \csc \alpha) - \frac{P}{2} \cot(\alpha) \right) \quad (6)$$

where M_w is the measurement over the wires, α is the half angle, P is the pitch and w is the mean diameter of the wires.

It is customary to use “best-size” wires (those that touch at the pitch line) and to assume the half angle and pitch are nominal values. For a 10 pitch 60-degree thread, the equation becomes

$$PD = M_w - \left(w(1 + 2) - \frac{0.1}{2} (\sqrt{3}) \right)$$

$$PD = M_w - (3w - 0.086603)$$

The term ($3w$ to 0.086603) is the “C” correction; it is the value to be subtracted from the measurement over the wires to get the pitch diameter. The “C” correction number is placed on the set of calibrated thread wires.

In our uncertainty budget we want to know how much the uncertainty of the diameter of the wires contributes to the total pitch diameter uncertainty. It is obvious in this case that a change of one microinch in the wire diameter will cause a change in the pitch diameter of three microinches. The “3” is called the sensitivity coefficient; it is the multiplier of the input quantity that will cause a change in the output quantity (pitch diameter in our case).

In some cases the input quantity that causes a change in the output quantity may not be in the same units as the output quantity. In that case the sensitivity coefficient will be a term that we multiply the input quantity by to get the correct units. An example is the change in length caused by a change in temperature. In a previous example, we calculated the change in the text accompanying the budget to get the value to put in the budget. The GUM tells us to get the sensitivity coefficient by taking the partial derivative of the equation modeling the measurement process. An easier way to determine the sensitivity coefficient is by numerical analysis, that is, we change the input variable a small amount and observe how much the output variable changes.

We will examine an uncertainty budget for a pitch diameter measurement. It is an example where it is obvious that if you change the input variable (mean wire diameter) one microinch, it will cause a change in the output variable (pitch diameter) of three microinches.

Sample Uncertainty Budget for the Measurement of Thread Pitch Diameter

Introduction

In this example we will develop an uncertainty budget to calculate the uncertainty of the measured value of the pitch diameter of a Class W 2½-10 UNC master external thread gage. The measurements were made in a temperature-controlled room using a measuring machine with resolution of one microinch to compare a gage block to the “measurement-over-wires” of the plug gage. The measurements were made in a room controlled to $20^{\circ}\text{C} \pm 1.0^{\circ}\text{C}$.

Method

The pitch diameter was measured as specified in the ASME B1.2 Standard.

Repeatability

In this example, 30 readings were taken to get a value for the repeatability of the measurement by going through the complete set-up and measuring process 30 times. The calculated standard deviation of the 30 readings was 16 microinches. Because this value was obtained by statistical means during the measurement process, the value is listed as a type A uncertainty estimate.

Uncertainty in “C” Correction

The uncertainty for the set of calibrated thread wires is 20 microinches. The 20 microinches was taken from a calibration certificate that gave the uncertainty at the 95 percent confidence level ($k=2$). One standard uncertainty equals 10 microinches. The pitch diameter is calculated from the equation

$$\text{PD} = M_w - \text{“C” correction.}$$

The “C” correction equals $(3w - 0.086603)$. The uncertainty in the “C” correction is three times the uncertainty in the wire diameter when measuring the pitch diameter of 60-degree threads; therefore, the uncertainty due to the wires is 30 microinches.

Scale Error

We are not using any corrections from the calibration report for the scale on the machine used to make the measurements. We only know that the worst value for the scale error is 20 microinches. We assume a rectangular distribution; that is, the error is equally likely to be any value between 0 and 20 μin . We divide by $\sqrt{3}$ to convert a rectangular distribution to one standard uncertainty.

Note: Most likely, the scale error is a linear function increasing with the length. If the machine calibration had given a graph showing the errors at each position, the measurement could have been corrected for the machine error. The uncertainty due to the scale would have been less than we are using in this uncertainty budget.

We could have eliminated most of the scale error by using a gage block combination the same size as the “measurement over wires” value.

Error in Force Setting

The Appendix in the ASME B1.2 Standard that describes how to measure pitch diameter states the force should be within 10% of the specified value. The machine has been calibrated and the force was checked at that time. We will assume the force is within 10% of the correct value when measuring the pitch diameter. We calculate the difference between the wire deformation at a 2.25 lb. force and a 2.75 lb. force as 6.1 μin using the Hertzian equations given in the Puttock and Thwaite paper, *Elastic Compression of Spheres and Cylinders at Point and Line Contact* (see References). Again we assume a rectangular distribution; we assume the force is equally likely to be any value between 2.25 and 2.75 pounds and we divided 6.1 μin by $\sqrt{3}$ to get 3.5 μin for one standard uncertainty.

Errors Due to Temperature

The coefficient of thermal expansion (α) of steel is approximately $11.5 \times 10^{-6}/^{\circ}\text{C}$. If we assume the thread gage is the same temperature as the measuring machine and gage block, the change in diameter of the gage due to temperature would be the nominal diameter times the difference between the coefficients of thermal expansion of the scale and the gage, times the room deviation from the nominal 20°C, or

$$\Delta L = (2.5 \text{ in})(1.0^{\circ}\text{C})(3.5 \times 10^{-6}/^{\circ}\text{C}) = 8.8 \mu\text{in}.$$

We estimated that the difference in the coefficient of thermal expansion between the scale and the gage could be as large as $3.5 \times 10^{-6}/^{\circ}\text{C}$. The temperature may be anywhere between 19°C and 21°C so we assume a rectangular distribution and divide by $\sqrt{3}$ to get one standard uncertainty of 5.05 microinches.

Note: The uncertainty due to the error in the thermometer of $\pm 0.02^{\circ}\text{C}$ degrees is considered to be insignificant.

The above is assuming the gages are the same temperature as the machine. However, it is more reasonable to assume that, due to handling, the gages might be as much as 0.25°C apart. A 0.25°C variation would cause an error of 7.2 μin , which, if we again assume a rectangular distribution, gives 4.2 μin for the standard uncertainty.

$$\Delta L = (2.5 \text{ in})(0.25^{\circ}\text{C})(11.5 \times 10^{-6}/^{\circ}\text{C}) = 7.2 \mu\text{in}.$$

Parallelism of the Anvils

The calibration report states that the out-of-parallelism of the anvils of the machine does not exceed 10 μin . Because we are measuring in the center of the anvils, we assume that the contribution from the anvils will not cause an error greater than half the maximum amount (5 μin). The actual parallelism may be any value between 0 and 10 μin , so we assume a rectangular distribution and divide 5 μin by $\sqrt{3}$ to get one standard uncertainty.

Error Caused by Assuming Half Angle and Pitch are Nominal

Because we used the best wire size to get the pitch diameter, there is no correction due to errors in the half angle. The specifications for the allowable deviation from nominal pitch for a Class W gage of this size is 0.0002 in. A change of one unit in the pitch, P, causes a change of 0.866 units in the pitch diameter. The actual error in pitch may be anywhere between ± 0.0002 in of nominal. Experience has shown that the error in pitch (between any two threads) is usually much less than the total pitch error. We will estimate it to be no more than one half of the total pitch error or 100 μin , which when multiplied by the sensitivity coefficient gives 87 μin . We will assume a normal distribution with all errors lying within the three-sigma limit. We then divide by 3 to obtain the standard uncertainty of 29 μin .

Table 6. Budget for Pitch Diameter Measurement of 2.5 in External Master Thread Gage

Source of Uncertainty	Estimate (μin)	Type	Degrees of Freedom	Distribution	Divisor	Sensitivity Coefficient	Standard Uncertainty (μin)	Variance (μin^2)
Repeatability	16	A	29	N	1	1	16	256
"C" correction	20	B	∞	N	2	3	30	900
Scale error	20	B	∞	R	$\sqrt{3}$	1	11.5	132
Force setting	6.1	B	∞	R	$\sqrt{3}$	1	3.5	12
Uncertainty in CTE	8.8	B	∞	R	$\sqrt{3}$	1	5.1	26
Part/master temperature difference	7.2	B	∞	R	$\sqrt{3}$	1	4.2	18
Parallelism of anvils	5	B	∞	R	$\sqrt{3}$	1	2.9	8.4
Error due to assuming pitch is within tolerance	100	B	∞	N	3	0.87	29	841
Summation	2193							
Combined standard uncertainty u_c	47							
Expanded uncertainty, $U = 2u_c$	94							

Sample Uncertainty Budget for the Major Diameter of a 20 Inch External Thread Gage

Table 7 shows a sample uncertainty budget for the measurement of the major diameter of a 20-inch external thread gage. The gage was measured using a measuring machine in a room controlled to $20^{\circ}\text{C} \pm 2^{\circ}\text{C}$. The maximum temperature difference between the part and gage block used to set the measuring machine was 0.2°C .

Repeatability

The repeatability was determined from 20 readings taken on the gage.

Gage Block

The machine was set with a 20 in gage block and the reading was corrected for the reported deviation of the block from the 20 in nominal. The value was taken from the calibration report.

Scale Error

The maximum uncertainty of the scale was taken from the calibration report of the measuring machine.

Force Setting

While the force setting is important in pitch diameter measurements, the effect is negligible for a major diameter measurement.

Uncertainty in the Coefficient of Expansion

In previous examples we listed the calculation in the text with the equation

$$\Delta L = L * \Delta T * \Delta \alpha,$$

where L is the nominal length, ΔT is the tolerance for the room temperature, and $\Delta \alpha$ is an estimate of the possible difference in the thermal coefficients of expansion between the master and the test item. Our estimate is $\Delta \alpha$, so the sensitivity coefficient is $L * \Delta T$, or $40 \mu\text{in}/^{\circ}\text{C}$.

We can put this information in the uncertainty budget without any text if we include the sensitivity coefficient as we have done in Table 7.

Part/Master Temperature Difference

For this uncertainty component of the budget, in previous examples we used the equation

$$\Delta L = L * \Delta T * \alpha$$

where ΔT is the possible difference in the temperature between the part and the master. Our estimate is ΔT , so the sensitivity coefficient is $L * \alpha$, or $230 \mu\text{in}/^{\circ}\text{C}$. We can put this information in the uncertainty budget without any text if we include the sensitivity coefficient as we have done in Table 7.

Parallelism of Anvils

The possible error due to the parallelism of the anvils is the same as it was for pitch diameter. In this case it is not a significant contributor to the expanded uncertainty. **Table 7. Budget for Major Diameter Measurement of 20-Inch External Master Thread Gage**

Source of Uncertainty	Estimate	Type	Distribution	Degrees of Freedom	Divisor	Sensitivity coefficient	Standard Uncertainty (μin)	Variance (μin ²)
Repeatability	100 μin	A	N	19	1	1	100	10000
Gage block	20 μin	B	N	∞	2	1	10	100
Scale error	20 μin	B	R	∞	√3	1	11.5	132
Force setting	0	B	R	∞	√3	1	0	0
Uncertainty in CTE	1.5/°C	B	R	∞	√3	40 μin/°C	34.6	1197
Part/master temperature difference	0.2°C	B	R	∞	√3	230 μin/°C	26.6	708
Parallelism of anvils	5 μin	B	R	∞	√3	1	2.89	8
Summation	12145							
Combined standard uncertainty (u _c)	110							
Expanded uncertainty U = 2u _c	220							

We could also have obtained the standard uncertainty for the temperature effects in the same manner as we have done in previous uncertainty budgets.

Sample Uncertainty Budget for the Calibration of an Autocollimator

In Table 8 we show an uncertainty budget for the calibration of an autocollimator. An autocollimator with a range of 10 minutes is calibrated by placing a mirror on a 10-inch sine plate and comparing the autocollimator readings to the angles set by the sine plate. Among other things, the uncertainty of the measurements is a function of how well we know the spacing between the rolls of the sine plate, l , and how well we know the length of the gage blocks, x , used to set the angle.

Repeatability

As shown in the budget, the repeatability was obtained by taking the standard deviation of 30 independent readings.

Gage Blocks

The value of the gage block combination is known to within $\pm 5 \mu\text{in}$. The effect on the angle was found by computing the sensitivity coefficient (the partial derivative of the angle with respect to the gage block stack, $\partial\theta/\partial x$). It is computed from the equation $\sin(\theta) = x/l$, or $\theta = \sin^{-1}(x/l)$. The sensitivity coefficient is $\frac{\partial\theta}{\partial x} = \frac{1}{\sqrt{l^2 - x^2}}$.

The estimate of the uncertainty is divided by $\sqrt{3}$, then multiplied by the sensitivity coefficient to get the standard uncertainty. We also divide by 4.85 to convert from microradians to seconds.

Sine Plate Length

The uncertainty in the angle generated with a sine plate is caused by uncertainty in the height of the gage block stack we just examined and also by uncertainty in the length of the sine plate itself. The uncertainty in the length of the 10-inch plate is specified to be 200 μin . The effect on the angle was found by computing the sensitivity coefficient (the partial derivative of the angle with respect to the sine plate length, $\partial\theta/\partial l$). It is computed from the equation $\sin(\theta) = x/l$, or $\theta = \sin^{-1}(x/l)$ where x is the height of the gage blocks and l is the length of the sine plate.

The sensitivity coefficient is $\frac{\partial\theta}{\partial l} = \frac{-x}{l\sqrt{l^2 - x^2}}$. The estimate of the uncertainty is divided by $\sqrt{3}$ then multiplied by

the sensitivity coefficient to get the standard uncertainty. We also divide by 4.85 to convert from microradians to seconds.

Estimating Sensitivity Coefficients

The GUM allows for alternative methods to estimate sensitivity values. For example, instead of taking partial derivatives, the measurement equation for angle [$\theta = \sin^{-1}(x/l)$] could be solved for a particular gage block height, x and then solved for $x \pm 5\mu\text{in}$. l would be 10 inches in both calculations. The difference between the two calculations is the uncertainty in angle caused by the gage block uncertainty. A similar set of calculations could then be performed for $l = 10$ inches and then for $l = 10.0002$ inches, with the same x value used for both calculations. The difference is the uncertainty in angle caused by the sine plate length uncertainty.

Alternatively, a table of sine plate “constants” could be used to find the change in gage block height needed to cause a change of one minute (60”) at the nominal angle. The height change should be converted to microinches and the sensitivity in seconds per inch can be calculated. (See Appendix B.)

Other analytical and approximation methods could be used. Unfortunately, for this budget, somewhat complicated calculations cannot be avoided. No matter what approach is used to determine sensitivity coefficients, it is necessary to very carefully to keep track of units.

For illustration, the budget in Table 8 has equations shown in the table. The calculations were made for a nominal angle of 10 minutes of arc.

Geometry

The table does not have perfect geometry. A worst case error of 0.1 second was estimated due to imperfect geometry of the table.

Temperature

Even though the temperature may vary from the nominal 20°C, a uniform change in the temperature will not change the angle.

Table 8. Uncertainty Budget for Measurement of an Autocollimator

Source of Uncertainty	Estimate	Type	DOF	Distribution	Divisor	Sensitivity coefficient	Standard Uncertainty (sec)	Variance μ^2
Repeatability	0.1 sec	A	29	N	1	1	0.1	0.01
Uncertainty in gage block combination	5 μ in	B	∞	R	$\sqrt{3}$	$\frac{1}{4.85\sqrt{l^2 - x^2}}$	0.1	0.01
Sine plate length	200 μ in	B	∞	R	$\sqrt{3}$	$\frac{x}{4.85(l)\sqrt{l^2 - x^2}}$	0.01	0.0001
Geometry of table	0.2 sec	B	∞	R	$\sqrt{3}$	1	0.12	0.01
Temperature	2°C	B	∞	R	$\sqrt{3}$	0	0	0
Summation	0.0301 sec ²							
Combined standard uncertainty (u_c)	0.17 sec							
Expanded uncertainty U ($k=2$)	0.35 sec							

A difference in this uncertainty budget from previous ones is that the change in angle caused by a change in the length of the sine plate is not linear. For small angles, the uncertainty caused by not exactly knowing the distance of the bar is negligible. At 45 degrees, the same uncertainty in the length of the sine plate would cause an error of 4 seconds in the angle. If done numerically, it is necessary to calculate the sensitivity coefficient for each angle measured when using a sine plate.

When k Does Not Equal 2

In most cases we report the uncertainty at the 95% confidence level using a coverage factor of $k=2$, but this assumes a normal distribution. If we combine several rectangular distributions, the result approaches a normal distribution. But when we have an uncertainty budget that contains only one source, and it is a rectangular distribution, then for a 95% confidence level, the coverage factor is less than 2.

The Student's t -table shown in Appendix F applies only to a normal distribution. By contrast, if the standard uncertainty is described by a rectangular distribution, as in this example, with standard deviation $\sigma = a/\sqrt{3}$, where a is the half-width of the distribution, then the level of confidence p is 57.75% for $k_p=1$; 95% for $k_p=1.65$; 99% for $k_p=1.71$; and 100% for $k_p = \sqrt{3} \approx 1.73$. The rectangular distribution is "narrower" than the normal distribution in the sense that it is of finite extent and has no "tails".

Sample Uncertainty Budget for the Calibration of a One Inch Micrometer

The case of calibrating a (0 to 1) inch digital micrometer illustrates this situation. The dominant source of the uncertainty is the resolution which is a rectangular distribution. The measurements were made using Grade 1 gage blocks in a temperature-controlled laboratory. The maximum variation in temperature from 68°F is $\pm 1^\circ\text{F}$.

The gage blocks and micrometer were allowed to stabilize, so the difference between the two is estimated to be less than 0.1°F.

Repeatability

A skilled operator obtained 0 repeatability.

Master

The actual values of the gage blocks were used. The uncertainty in the calibration was 4 μin which was taken from the calibration report which reported a 95% confidence level or $k=2$.

Resolution

The resolution of the micrometer is 100 μin. To obtain the standard uncertainty we divide the resolution by two and then by the square root of three, or $100/2\sqrt{3} = 29$ μin.

Uncertainty in Coefficient of Thermal Expansion

The standard uncertainty is calculated from the equation $\Delta L = L * (68^{\circ}\text{F} - T) * \Delta\alpha$, or $\Delta L = (1 \text{ in}) (1^{\circ}\text{F})(0.9 \times 10^{-6}/^{\circ}\text{F}) = 0.9$ μin.

Uncertainty in Temperature Difference between Master and Test

The uncertainty caused by the temperature difference between the master and the test is given by the equation $\Delta L = L * \Delta T * \alpha$, or $\Delta L = (1 \text{ in}) (0.1^{\circ}\text{F})(6.5 \times 10^{-6}/^{\circ}\text{F}) = 0.6$ μin.

Table 9. Uncertainty Budget for Calibrating a (0 to 1) Inch Digital Micrometer

Uncertainty Source	Estimate (μin)	Type	Distribution	Divisor	Standard Uncertainty (μin)	Variance (μin ²)
Gage blocks	4	B	Normal	2	2	4
Resolution	100	B	Rectangular	$2\sqrt{3}$	29	841
Uncertainty of CTE	0.9	B	Rectangular	$\sqrt{3}$	0.52	0.27
Master/part temperature difference	0.6	B	Rectangular	$\sqrt{3}$	0.35	0.12
Summation	845					
Combined Standard Uncertainty	29					
Expanded Uncertainty (k=1.65)	48					

An examination of the uncertainty budget shows that it consists of only one significant source, the resolution, which is a rectangular distribution. In fact, this single contributor is so large compared to the others that all of the other contributors could be safely ignored and the resolution could be entered as the sole uncertainty contributor in the budget. In cases like this we could express the expanded uncertainty as a function of the resolution: $U = (1.65/(2\sqrt{3}))*R \approx 0.48R$.

For the purposes of CMC, it is acceptable to separate out the device contribution. In this case, separating out the resolution contribution would produce a CMC of 4.2 μm with a statement such as “This value does not include the contribution from the device under test. This contribution is expected to be 0.6R.”

Appendix A. Student's *t*-Distribution

Value of $t_p(v)$ from the *t*-distribution for degrees of freedom v that defines an interval $-t_p(v)$ to $+t_p(v)$ that encompasses the fraction p of the distribution.

Degrees of freedom v	Fraction “p” in percent					
	68.27 ^(a)	90.00	95.00	95.45 ^(a)	99.00	99.73 ^(a)
1	1.84	6.31	12.71	13.97	63.66	235.8
2	1.32	2.92	4.30	4.53	9.92	19.21
3	1.20	2.35	3.18	3.31	5.84	9.22
4	1.14	2.13	2.78	2.87	4.60	6.62
5	1.11	2.02	2.57	2.65	4.03	5.51
6	1.09	1.94	2.45	2.52	3.71	4.90
7	1.08	1.89	2.36	2.43	3.50	4.53
8	1.07	1.86	2.31	2.37	3.36	4.28
9	1.06	1.83	2.26	2.32	3.25	4.09
10	1.05	1.81	2.23	2.28	3.17	3.96
11	1.05	1.80	2.20	2.25	3.11	3.85
12	1.04	1.78	2.18	2.23	3.05	3.76
13	1.04	1.77	2.16	2.21	3.01	3.69
14	1.04	1.76	2.14	2.20	2.98	3.64
15	1.03	1.75	2.13	2.18	2.95	3.59
16	1.03	1.75	2.12	2.17	2.92	3.54
17	1.03	1.74	2.11	2.16	2.90	3.51
18	1.03	1.73	2.10	2.15	2.88	3.48
19	1.03	1.73	2.09	2.14	2.86	3.45
20	1.03	1.72	2.09	2.13	2.85	3.42
25	1.02	1.71	2.06	2.11	2.79	3.33
30	1.02	1.70	2.04	2.09	2.75	3.27
35	1.01	1.70	2.03	2.07	2.72	3.23
40	1.01	1.68	2.02	2.06	2.70	3.20
45	1.01	1.68	2.01	2.06	2.69	3.18
50	1.01	1.68	2.01	2.05	2.68	3.16
100	1.005	1.660	1.984	2.025	2.626	3.077
∞	1.000	1.645	1.960	2.000	2.576	3.000
^(a) For a quantity z described by a normal distribution with expectation μ_z and standard deviation σ , the interval $\mu_z \pm k\sigma$ encompasses $p = 68.27, 95.45$, and 99.73 percent of the distribution for $k = 1, 2$, and 3 respectively.						

Appendix B. Sensitivity Coefficient Example

The sine plate formula is:

$$\theta = \sin^{-1}(x/l)$$

The relationships between the angle in degrees, minutes, and seconds are:

$$\text{Minutes (')} = \text{Degrees} \times 60$$

$$\text{Seconds (')} = \text{Minutes} \times 60$$

The sine plate length, l , is 10 in.

The gage block height is x . Let x be 0.03000 in.

$$\theta_1 = \sin^{-1}(0.03000/10) = 0.171888^\circ = 10.31326' = 618.800''$$

Let us change the height by a gage block uncertainty of 5 μin and recalculate.

$$\theta_2 = \sin^{-1}(0.030005/10) = 0.171916^\circ = 10.31498' = 618.898''$$

The change in angle produced by a 5 μin change in gage block height is:

$$\theta_2 - \theta_1 = 0.09848''$$

The change per microinch is:

$$0.09848/5 = 0.02 \text{ seconds per microinch}$$

This is the gage block height sensitivity coefficient value for a nominal angle of 10'.

(For the standard uncertainty in gage block combination in Table 8, we divided the 5 μin value by $\sqrt{3}$ and then multiplied times 0.02 to get 0.06, which was rounded up to 0.1 second.)

To find the sine plate length sensitivity coefficient, the same process is used, except holding x constant and calculating the angle first for a sine plate length of 10 in and then for 10 in plus 200 μin .

References

- JCGM 100:2008, *Evaluation of measurement data - Guide to the Estimation of Uncertainty in Measurement*.
- ASME B1.1-2003 *Unified Inch Screw Threads (UN and UNR Thread Form)*
- ASME B1.2-1983 *Gages and Gages for Unified Inch Screw Threads*
- ASME B1.3-2007 *Screw Thread Gaging Systems for Acceptability Inch and Metric Screw Threads (UN, UNR, UNJ, M, and MJ)*
- Barry N. Taylor and Chris E. Kuyatt, *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note 1297 (1994).
- D.A. Swyt, *Uncertainties in Dimensional Measurements Made at Nonstandard Temperatures*, J. Res. Natl. Inst. Stand. Technol., 99 (1), 31-34 (1994).
- ILAC P14:09/2020 *ILAC Policy for Uncertainty in Calibration*
- ISO/TS 14253-2: 2002: *Geometrical Product Specifications (GPS) -- Inspection by measurement of workpieces and measuring equipment -- Part 2: Guide to the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification*.
- NCSL, *Determining and Reporting Measurement Uncertainties*, Recommended Practice RP12 (2013).
- M.J. Puttock and E.G. Thwaite, *Elastic Compression of Spheres and Cylinders at Point and Line Contact*, National Standards Laboratory Technical Paper No. 25, Commonwealth Scientific and Industrial Research Organization (1969).
- J.G. Salsbury, *Uncertainty of the Coordinate Measurement Process Due to Unwanted Asperities*, American Society for Precision Engineering, Volume 14 (1996).
- S.D. Phillips, B.R. Borchardt, W.T. Estler, *The Estimation of Measurement Uncertainty of Small Circular Features Measured by CMMs*, NISTIR 5698; National Institute of Standards and Technology, Gaithersburg, MD, (1995).
- T. Doiron and J. Stoup, *Uncertainty and Dimensional Calibrations*, J. Res. Natl. Inst. Stand. Technol., 102 (6), 647-676, 1997.
- UKAS M3003, *The Expression of Uncertainty and Confidence in Measurement*, 2019.

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